

Contents

Acknowledgements	v
Abstract (English/Deutsch)	ix
List of Publications	xiii
List of Presentations	xv
1 Microresonator based optical frequency combs	1
1.1 Optical frequency combs	2
1.1.1 Application of frequency combs	3
1.1.2 Generation of optical frequency combs in mode-locked lasers	3
1.1.3 Self-referencing and stabilization of optical frequency combs	4
1.2 Dielectric nonlinear microresonators	6
1.2.1 Basic microresonator characteristics	6
1.2.2 Microresonators used in this work	12
1.3 Microresonator based optical frequency combs	16
1.3.1 Application of microresonator based combs	17
1.3.2 Kerr-nonlinearity and nonlinear parametric frequency conversion	18
1.3.3 Thermal and Kerr-nonlinear resonance shift and related concepts	22
1.3.4 Frequency comb generation via four-wave mixing	26
1.3.5 Competing non-linear processes	31
1.3.6 Setup for frequency comb generation	33
1.3.7 State-of-the-art in microresonator based frequency combs	33
2 Formation dynamics and noise in microresonator based frequency combs	39
2.1 Noise in broadband spectra with many comb lines	40
2.1.1 Generation of an octave-spanning spectrum in a SiO ₂ toroid	40
2.1.2 Characterization of frequency noise in microresonator based combs	43
2.1.3 Observation of noise in a SiO ₂ toroid	44
2.1.4 The universal problem of noise in microresonator based combs	46
2.1.5 Linking microresonator based comb formation dynamics and noise	47

2.2	Microresonator based comb formation	49
2.2.1	Generation of subcombs	49
2.2.2	Commensurability of subcombs	50
2.3	Emergence of noise in microresonator based combs	56
2.3.1	Multiple and broad RF beatnotes	56
2.3.2	Intensity noise at low-frequencies	57
2.3.3	Line-by-Line reconstruction of microresonator combs	60
2.4	Transition to low noise	62
2.4.1	The role of dispersion and resonance width	63
2.5	Discussion and conclusion	64
3	Mid-infrared frequency combs based on microresonators	67
3.1	The mid-infrared regime	67
3.2	Microresonator based mid-infrared combs	68
3.2.1	Crystalline magnesium fluoride as a resonator material for the mid-infrared	68
3.2.2	Generation of mid-infrared frequency comb	70
3.2.3	Phase-noise characterization	73
3.2.4	Proof-of-principle direct comb spectroscopy experiment	75
3.3	Discussion and conclusion	76
4	Temporal soliton formation in microresonators	79
4.1	Temporal dissipative Kerr-cavity solitons	80
4.2	Evidence of soliton formation in a microresonator	80
4.2.1	Microresonator characteristics	81
4.2.2	Kerr-nonlinear and thermal resonance shifts and stability	82
4.2.3	Observation of discrete steps in the resonator transmission	83
4.3	Theoretical description of temporal dissipative solitons in microresonators	85
4.3.1	Numerical simulation	85
4.3.2	Analytic description	86
4.4	Experimental generation of temporal dissipative solitons in a micro- resonator	89
4.4.1	Laser tuning method	89
4.4.2	Stability of the soliton states	91
4.4.3	Spectral and temporal characterization	92
4.5	Resonator mode structure and soliton formation	97
4.5.1	Global mode structure	97
4.6	Detailed investigation of selected mode families	99
4.7	Spectral broadening and supercontinuum generation	102
4.8	Discussion and conclusion	106
5	Outlook	109

A Theory of microresonator based frequency comb generation	111
A.1 Optical coupling to microresonators	111
A.2 Frequency comb formation in microresonators	113
A.2.1 Nonlinear coupled mode equations	113
A.2.2 Three mode system	115
A.2.3 Nonlinear resonance shift, effective detuning and optical bistability	116
A.2.4 Threshold pump power	118
A.2.5 Emergence of the first mode	118
A.2.6 Emergence of secondary comb lines	119
B Noise consideration in a SiO₂ toroid	123
C Theory of temporal dissipative solitons in microresonators	129
C.1 Analytical description of solitons in a microresonators	129
C.2 Limit conditions for solitons in microresonators	131
C.3 Analytical description of steps in the intracavity power	132
C.4 Optical spectrum and temporal width of solitons in a microresonator .	132
C.5 Soliton mode-locking in lasers vs. soliton formation in microresonators	133
D Experimental methods	135
D.1 Frequency comb calibrated broadband laser scan	135
D.1.1 Setup and principle	135
D.1.2 Data post-processing	136
D.1.3 Limitation of the method	139
D.1.4 Measurement of resonator dispersion and comb reconstruction	139
D.2 Measurement of microcavity resonance width	141
Bibliography	142
Curriculum vitae	159