

Contents

Acknowledgements	v
Abstract (English/Français)	vii
List of figures	xvi
List of tables	xxi
Introduction	1
Motivation	1
Outline and contributions of the thesis	4
1 Classical extreme value theory	7
1.1 Univariate extreme value theory	7
1.1.1 Asymptotic distribution for linearly renormalized maxima	7
1.1.1.1 Basic results	7
1.1.1.2 Maximum domains of attraction	12
1.1.1.3 Extension to stationary series	15
1.1.1.4 Inference	18
1.1.2 Point process approach	20
1.1.2.1 Definitions and basic results	21
1.1.2.2 Point process of exceedances	23
1.1.2.3 Extension to stationary series	25
1.1.2.4 Interpretation and estimators for the extremal index	28
1.1.2.5 Inference	30
1.2 Multivariate extreme value theory	33
1.2.1 Componentwise maximum approach	35
1.2.1.1 Multivariate extreme value distributions and max-stability	35
1.2.1.2 The exponent measure	36
1.2.1.3 Spectral representation for multivariate extreme value distributions	37
1.2.1.4 Pickands' dependence function	37

1.2.1.5	Parametric models	38
1.2.1.6	Inference	41
1.2.2	Point process approach	43
1.2.2.1	Basic results and connection to componentwise maxima	43
1.2.2.2	Inference methods	45
1.2.3	Copula modeling for multivariate extremes	55
1.2.4	Asymptotic independence	58
1.2.4.1	Ledford & Tawn model	60
1.2.4.2	Inverted multivariate extreme value distributions . . .	62
1.2.5	Measures of extremal dependence	62
1.2.5.1	Extremal coefficient θ_D	63
1.2.5.2	Coefficient of tail dependence η	64
1.2.5.3	Coefficients χ and $\bar{\chi}$	64
1.3	Summary	66
2	Geostatistical modeling of extremes in space and time	67
2.1	Fundamentals of spatial random processes	69
2.1.1	Definitions and notation	69
2.1.2	Important properties	70
2.1.2.1	Stationarity	70
2.1.2.2	Isotropy	71
2.1.2.3	Ergodicity	72
2.1.3	Covariance functions and variograms	73
2.1.3.1	Basic notions and classical models	73
2.1.3.2	Space-time correlation functions and related properties	77
2.2	Hierarchical models for extremes	82
2.2.1	Cooley et al.'s model	82
2.2.2	Sang–Gelfand model	83
2.3	Max-stable processes	84
2.3.1	Generalities	84
2.3.2	Stationary parametric models	89
2.3.2.1	Smith model	90
2.3.2.2	Schlather model	92
2.3.2.3	Brown–Resnick model	94
2.3.2.4	Extremal- t model	97
2.3.2.5	Other models	97
2.3.3	Models based on α -stable random effects	100
2.3.4	Max-stable processes for threshold exceedances	101
2.4	Asymptotic independence and related models for spatial extremes . .	102
2.4.1	Gaussian copula model	103

2.4.2	Inverted max-stable processes	103
2.4.3	Hybrid models	104
2.5	Measures of extremal dependence	105
2.6	Inference for extremal models	107
2.7	Application	108
2.8	Summary	111
3	Inference based on composite likelihoods	115
3.1	Composite likelihoods	116
3.1.1	Definitions	116
3.1.2	Marginal likelihoods	117
3.1.3	Asymptotics	117
3.1.4	Model comparison	118
3.1.5	Estimation of the asymptotic variance	119
3.2	Pairwise likelihood for spatial extremes	120
3.2.1	Componentwise maxima	120
3.2.2	Threshold exceedances	120
3.3	Efficiency of pairwise likelihoods	121
3.3.1	Previous work and weighting strategy	122
3.3.2	Gaussian models	124
3.3.2.1	Theoretical results for AR(1) and MA(1) models	124
3.3.2.2	Simulation study for ARMA models	133
3.3.2.3	Optimal weights for Gaussian processes	138
3.3.3	Max-stable models	144
3.3.3.1	Maximum likelihood estimation for the logistic model	144
3.3.3.2	Simulation study for the Schlather model with random set	156
3.4	Summary	160
4	Composite likelihood estimation for the Brown–Resnick process	163
4.1	Brown–Resnick process constructed from fractional Brownian motions	164
4.1.1	Definition and properties	164
4.1.2	Inference based on pairwise likelihood	166
4.2	Derivation of the likelihood	166
4.2.1	Exponent measure	166
4.2.1.1	Case $D = 3$	166
4.2.1.2	Case $D > 3$	169
4.2.2	Density	170
4.3	Efficiency gains of the triplewise likelihood approach	173
4.3.1	Inference based on triplewise likelihood	173
4.3.2	Simulation study	173

4.3.2.1	Comparison of efficiencies for increasing n and fixed S	174
4.3.2.2	Comparison of efficiencies for increasing S and fixed n	175
4.3.2.3	Further comments	176
4.4	Inference using the occurrence times of extreme events	177
4.4.1	Stephenson–Tawn likelihood	178
4.4.2	Relative efficiencies of marginal likelihood estimators	180
4.5	Discussion and extensions	182
5	Real case study: Space-time modeling of extreme rainfall	185
5.1	Threshold modeling for extremes	185
5.1.1	Marginal modeling	186
5.1.2	Dependence modeling based on max-stable processes	187
5.2	Inference	188
5.2.1	Censored pairwise likelihood approach	188
5.2.2	Asymptotics under mixing conditions	189
5.2.3	Variance estimation	195
5.3	Data analysis	195
5.3.1	Exploratory analysis	196
5.3.1.1	Description of the dataset	196
5.3.1.2	Marginal distributions	198
5.3.1.3	Stationarity	202
5.3.1.4	Spatio-temporal dependence	204
5.3.2	Modeling of extremal dependence	206
5.3.2.1	Schlather model with random set	206
5.3.2.2	Alternative max-stable models	212
5.3.2.3	Asymptotic independence models	214
5.3.3	Model comparison	216
5.3.4	Summary	218
5.4	Discussion and perspectives	222
	Conclusion and future work	223
	Appendix	227
A	Performance of various estimators for the bivariate extreme-value logistic model	227
B	Pairwise margins for max-stable and asymptotic independence models	231
B.1	Max-stable models	231
B.1.1	Smith and Brown–Resnick models	232
B.1.2	Schlather model with or without random set	232

B.1.3 Extremal- t model	233
B.2 Asymptotic independence models	233
B.2.1 Gaussian copula model	233
B.2.2 Inverted max-stable models	234
B.2.3 Hybrid models	234
C Consistency and efficiency of pairwise and triplewise likelihood estimators for the Brown–Resnick process	237
D Performance of composite Stephenson–Tawn likelihood estimators for the Brown–Resnick process	241
E Additional diagnostic plots of extremal dependence for the rainfall data	245
F Computation of the volume of overlap $\delta(h_s, h_t)$	251
G Trivariate extremal coefficients for model (5.12)	255
H Simulation of the fitted max-stable model (5.12) in space and time	257
Bibliography	261
Index	281
Curriculum Vitae	287