

Contents

| | |
|--|-----------|
| Abstract | ix |
| 1 Introduction | 1 |
| 1.1 Outline and Contribution | 4 |
| 1.2 Publications | 5 |
| 1.3 Software | 6 |
| I Preliminaries | 7 |
| Notation | 9 |
| 2 Convex Optimization | 11 |
| 2.1 Basics on Convex Optimization | 11 |
| 2.1.1 Convex Problems with Linear Inequalities | 12 |
| 2.1.2 Conic Programming | 13 |
| 2.1.3 Symmetric Cones | 15 |
| 2.1.4 Self-Dual Embeddings | 18 |
| 2.2 Second-Order Methods for Convex Programming | 21 |
| 2.2.1 Newton's Method for Unconstrained Optimization | 21 |
| 2.2.2 Interior Point Methods | 25 |
| 3 Direct Methods for Solving KKT Systems | 39 |
| 3.1 Symmetric Indefinite Forms | 40 |
| 3.1.1 LDL Factorization | 40 |
| 3.1.2 Quasi-Definite Matrices | 41 |
| 3.2 Positive Definite Form | 42 |
| 3.2.1 Cholesky Factorization | 42 |
| 3.2.2 Low Rank Modifications of Matrix Factors | 43 |
| 3.3 Iterative Refinement | 46 |

| | | |
|-----------|--|-----------|
| 4 | Model Predictive Control | 49 |
| 4.1 | Linear MPC | 49 |
| 4.1.1 | Lyapunov Stability | 49 |
| 4.1.2 | Nominal MPC | 50 |
| 4.1.3 | Suboptimal MPC | 51 |
| 4.2 | Hybrid Models and MPC | 52 |
| 5 | Machine Learning | 55 |
| 5.1 | Classification | 55 |
| 5.1.1 | Support Vector Machines | 55 |
| 5.1.2 | (Adaptive) Boosting | 56 |
| 5.2 | Kernel Regression | 58 |
| II | Approximation of MPC Control Laws from Data | 59 |
| 6 | Certified Learning of MPC Control Laws | 61 |
| 6.1 | Introduction | 61 |
| 6.2 | Problem Setup | 63 |
| 6.3 | The Semi-Infinite Learning Problem | 63 |
| 6.4 | Derivation of a Tractable Learning Problem | 65 |
| 6.4.1 | Satisfying Input Constraints | 66 |
| 6.4.2 | Recursive Feasibility | 67 |
| 6.4.3 | Stability | 69 |
| 6.4.4 | Main Result | 72 |
| 6.5 | Numerical Example | 73 |
| 7 | Learning Decision Rules for Energy Efficient Building Control | 77 |
| 7.1 | Introduction | 77 |
| 7.2 | Hybrid Model Predictive Building Control | 79 |
| 7.2.1 | Hierarchical Control Structure | 79 |
| 7.2.2 | Hybrid MPC Formulation | 80 |
| 7.3 | Rule Extraction from Simulation Data | 82 |
| 7.3.1 | Support Vector Classification | 83 |
| 7.3.2 | AdaBoost | 85 |
| 7.4 | Simulation Results | 88 |
| 7.4.1 | Setup | 88 |
| 7.4.2 | Approximation Accuracy | 91 |

| | | |
|-------|-------------------------|----|
| 7.4.3 | Feature Selection | 92 |
| 7.4.4 | Closed-loop Performance | 94 |
| 7.5 | Conclusion | 96 |

III Fast Interior Point Methods for Embedded Optimization 97

| | | |
|----------|--|------------|
| 8 | Code Generation for Convex Multistage Problems | 99 |
| 8.1 | Introduction | 99 |
| 8.2 | Efficient Interior Point Solver for Multistage Problems | 102 |
| 8.2.1 | Problem Structure | 102 |
| 8.2.2 | Search Direction Computation in the General Case | 104 |
| 8.2.3 | Structure Exploitation for Common Problem Instances | 106 |
| 8.2.4 | Computational Results | 112 |
| 8.3 | Parallelization | 117 |
| 8.3.1 | Computational Model for Multi-Core Architectures | 118 |
| 8.3.2 | Linear Algebra Acceleration Using FPGAs | 119 |
| 8.4 | Code Generation | 124 |
| 8.4.1 | Supported Problem Class | 127 |
| 8.4.2 | Implementation Details | 128 |
| 8.4.3 | Benchmark Results for Generated Code | 130 |
| 8.5 | Application Examples | 132 |
| 8.5.1 | Robust Real-time MPC for Setpoint Tracking | 132 |
| 8.5.2 | Automatic Racing of 1:43 Scale RC Race Cars | 139 |
| 9 | A Second-order Cone Programming Solver for Embedded Systems | 145 |
| 9.1 | Introduction | 145 |
| 9.2 | Path-Following Infeasible Primal-Dual IPM | 147 |
| 9.2.1 | Central Path | 147 |
| 9.2.2 | Scalings | 148 |
| 9.2.3 | Search Directions | 150 |
| 9.3 | Efficient Search Direction Computation | 157 |
| 9.3.1 | Sparse LDL Factorization with Fixed Ordering | 157 |
| 9.3.2 | Structure Exploitation of Second-Order Cone NT-Scalings | 161 |
| 9.4 | Implementation Details | 167 |
| 9.4.1 | Initialization | 167 |
| 9.4.2 | Stopping Criteria | 169 |
| 9.4.3 | Implementation Overview | 170 |

| | | |
|-----------|---|------------|
| 9.5 | Examples and Benchmarks | 172 |
| 9.5.1 | Portfolio Optimization | 172 |
| 9.5.2 | DIMACS Challenge Tests | 173 |
| 9.5.3 | Soft-Constrained Tracking MPC with Stability Guarantees | 175 |
| 9.6 | Conclusion | 176 |
| 10 | Discussion and Outlook | 179 |
| IV | Appendix | 181 |
| A | Definitions and Facts from Convex Analysis | 183 |
| A.1 | Self-Dual Embeddings | 183 |
| A.2 | Theorem of Alternatives for Generalized Inequalities | 184 |
| B | Results and Formulations in Model Predictive Control | 185 |
| B.1 | Reference Tracking | 185 |
| B.2 | Multistage Problem Matrices for Robust Real-time Tracking Problem | 186 |
| B.2.1 | Cost function | 186 |
| B.2.2 | Equality constraints | 187 |
| B.2.3 | Inequality constraints | 188 |
| | Bibliography | 190 |
| | Curriculum Vitae | 203 |