

# Contents

<b>Abstract</b>	<b>i</b>
<b>Zusammenfassung</b>	<b>iii</b>
<b>Contents</b>	<b>v</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Anomalous dynamics . . . . .	1
1.2 Critical dynamics at phase transitions . . . . .	4
1.3 The dissipative central spin model . . . . .	7
1.4 Outline of this thesis . . . . .	9
<b>2 The quantum dot central spin model</b>	<b>13</b>
2.1 Semiconductor quantum dots . . . . .	13
2.2 The central electron spin . . . . .	15
2.2.1 Electron Zeeman interaction . . . . .	15
2.2.2 Continuous wave laser excitation . . . . .	16
2.2.3 Periodic pulsed laser excitation . . . . .	19
2.2.4 Dissipation by spontaneous emission . . . . .	23
2.2.5 Electron master equation in the absence of nuclear spins . . . . .	24
2.3 The nuclear environment . . . . .	26
2.3.1 Nuclear Zeeman interaction . . . . .	26
2.3.2 Dipolar interactions . . . . .	27
2.3.3 Quadrupolar interactions . . . . .	27
2.3.4 Collective nuclear spin states . . . . .	28
2.4 The electron–nuclei interactions . . . . .	31
2.4.1 Hyperfine interaction . . . . .	32
2.4.2 Non–collinear interaction . . . . .	38
2.5 Coupled dynamics in the quantum dot spin system . . . . .	39
2.5.1 Electron spin decoherence . . . . .	39
2.5.2 Dynamic nuclear polarization . . . . .	43
2.5.3 Summary of electron–nuclei feedback discussed in this thesis . . . . .	45
<b>3 Nuclear spin cooling using Overhauser field selective coherent population trapping</b>	<b>49</b>
3.1 Motivation . . . . .	49
3.2 Driven–dissipative electron–nuclei system . . . . .	51
3.3 Description of the coupled electron–nuclei dynamics . . . . .	53
3.3.1 Derivations . . . . .	53
3.3.2 Qualitative description of the coupled dynamics . . . . .	59

3.4	Nuclear spin cooling and spectroscopic signatures . . . . .	60
3.4.1	Homogeneous hyperfine coupling . . . . .	60
3.4.2	Inhomogeneous hyperfine coupling . . . . .	63
3.4.3	Suppression of $T_2$ decoherence . . . . .	66
3.4.4	Spectrum and density of states of the generalized Overhauser field . . . . .	66
3.4.5	Generalization to multiple nuclear spin species . . . . .	72
3.4.6	Nuclear spin decoherence . . . . .	74
3.5	Anomalous nuclear spin diffusion . . . . .	74
3.6	Conclusion . . . . .	79
<b>4</b>	<b>Real-time monitoring of Lévy flights in a single quantum system</b>	<b>81</b>
4.1	Introduction . . . . .	81
4.2	System . . . . .	82
4.3	Periodic optical excitation of the electron . . . . .	84
4.3.1	Derivations . . . . .	85
4.4	Coupled electron–nuclei dynamics . . . . .	92
4.4.1	Schrieffer–Wolff transformation . . . . .	93
4.5	Lévy flights in the nuclear environment . . . . .	97
4.6	Real-time monitoring of the Lévy flights . . . . .	99
4.7	Limitations of the scheme . . . . .	100
4.7.1	Modified trapping time distribution . . . . .	101
4.7.2	Physical origin of the limitations . . . . .	102
4.7.3	Electron dephasing . . . . .	103
4.7.4	Imperfect laser polarization . . . . .	104
4.7.5	Intrinsic limitation . . . . .	106
4.7.6	Displaced echo sequence . . . . .	106
4.8	Conclusion and outlook . . . . .	109
<b>5</b>	<b>Signatures of a dissipative phase transition controlled by a single quantum system</b>	<b>111</b>
5.1	Introduction . . . . .	111
5.2	System . . . . .	112
5.3	Master equation for the spin environment . . . . .	114
5.3.1	Adiabatic elimination of the excited central system state . . . . .	114
5.3.2	Adiabatic elimination of environmental spin coherences . . . . .	116
5.3.3	Tracing over the degrees of freedom of the central spin . . . . .	117
5.4	Signatures of criticality . . . . .	118
5.4.1	Realization in quantum dots . . . . .	119
5.4.2	Phase characterization by dynamic nuclear polarization . . . . .	119
5.4.3	Continuous change in the dynamic nuclear polarization . . . . .	120
5.4.4	Evolution of the nuclear distribution . . . . .	121
5.4.5	Multiple nuclear species . . . . .	122
5.4.6	Spectral gap . . . . .	126
5.4.7	Thermodynamic limit . . . . .	127
5.4.8	Constant hyperfine coupling . . . . .	130
5.5	Conclusion and outlook . . . . .	130
<b>6</b>	<b>Conclusion and outlook</b>	<b>133</b>

<b>List of figures</b>	<b>147</b>
<b>Curriculum vitae</b>	<b>152</b>
<b>Acknowledgments</b>	<b>155</b>