

# Contents

<b>Acknowledgements</b>	<b>iii</b>
<b>Summary</b>	<b>v</b>
<b>Zusammenfassung</b>	<b>vii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Motivation . . . . .	1
1.2 Rarefied gas dynamics . . . . .	4
1.2.1 Ideal gases . . . . .	4
1.2.2 Modelling methods . . . . .	5
1.3 Thermodynamically admissible 13 moment eqns. . . . .	7
1.3.1 Variables . . . . .	7
1.3.2 Evolution equations . . . . .	9
<b>2 Linear stability analysis</b>	<b>13</b>
2.1 Overview of a basic eigenvalue analysis . . . . .	13
2.2 Linear stability analysis of sound waves . . . . .	14
2.2.1 Linearization . . . . .	14
2.2.2 Plane waves . . . . .	16
2.3 Results . . . . .	19
2.3.1 Dispersion relation . . . . .	19
2.3.2 Linear stability in time . . . . .	19
2.3.3 Linear stability in space . . . . .	37
2.3.4 Dispersion and damping . . . . .	38
2.4 Summary . . . . .	43
<b>3 Shock tube</b>	<b>45</b>
3.1 Introduction . . . . .	45
3.2 Shock tube geometry . . . . .	46
3.3 Hyperbolic conservation laws . . . . .	49
3.3.1 Definition of hyperbolicity . . . . .	49
3.3.2 Hyperbolic region . . . . .	50
3.3.3 Nonconservative products . . . . .	53

3.4	Numerical method . . . . .	55
3.5	Improved numerical implementation. . . . .	58
3.6	Results . . . . .	60
3.6.1	Reference comparison . . . . .	60
3.6.2	Convergence . . . . .	64
3.6.3	Stability comparison . . . . .	71
3.6.4	Shock wave . . . . .	74
3.7	Summary . . . . .	80
<b>4</b>	<b>Boundary conditions</b>	<b>81</b>
4.1	Geometry of rarefied channel flow simulations . . . . .	82
4.2	Derivation of the physical conditions at the boundary . . . . .	86
4.2.1	The accommodation model . . . . .	86
4.2.2	Irreversible thermodynamics . . . . .	87
4.3	Treatment of the boundary conditions . . . . .	89
4.4	The numerical algorithm . . . . .	91
4.4.1	Eigenvector based approach . . . . .	91
4.4.2	Extrapolation algorithm at the boundary . . . . .	91
4.5	Results . . . . .	96
4.6	Summary . . . . .	103
<b>5</b>	<b>Conclusions and outlook</b>	<b>105</b>
<b>A</b>	<b>Appendix</b>	<b>107</b>
A.1	Boundary conditions for Grad 13 . . . . .	107
A.1.1	The Grad 13 equations in the channel geometry . . . . .	107
A.1.2	Grad 13 boundary conditions . . . . .	109
A.2	Heattransfer . . . . .	110
A.2.1	Problem setting . . . . .	110
A.2.2	Torrilhon's analysis . . . . .	111
A.2.3	Discussion . . . . .	111
A.2.4	Simulation setup . . . . .	113
	<b>Bibliography</b>	<b>119</b>