

# Contents

<b>Introduction</b>	<b>xiii</b>
<b>1. Preliminaries</b>	<b>1</b>
1.1. System parameters . . . . .	1
1.2. Regime . . . . .	1
1.3. Layer potentials . . . . .	2
1.4. Floquet Transform . . . . .	2
1.5. Quasi-periodic layer potentials . . . . .	3
1.6. Incident and scattered fields . . . . .	4
1.7. Notation . . . . .	4
<b>2. Minnaert Resonance of an Arbitrary Shaped Sub-wavelength Resonator</b>	<b>5</b>
2.1. Introduction . . . . .	5
2.2. The Minnaert resonance . . . . .	6
2.3. The point scatterer approximation . . . . .	13
2.4. The Minnaert resonance in two dimensions . . . . .	17
2.5. Numerical illustrations . . . . .	21
2.6. Concluding remarks . . . . .	23
<b>3. Perfect Absorption using Sub-wavelength Acoustic Resonators</b>	<b>25</b>
3.1. Introduction . . . . .	25
3.2. Statement of the problem . . . . .	26
3.3. Proof of Theorem 3.2.1. . . . .	31
3.3.1. Quasi-periodic Green's functions . . . . .	31
3.3.2. Quasi-periodic-Dirichlet layer potentials . . . . .	35
3.3.3. Equivalent formulation . . . . .	36
3.3.4. Asymptotic expansions . . . . .	39
3.3.5. Microscopic scattered field . . . . .	44
3.4. Numerical illustrations . . . . .	46
3.4.1. Implementation details . . . . .	47
3.4.2. Validation of the Minnaert frequency formula . . . . .	48
3.4.3. Dependence of the Minnaert resonance with respect to the geometry . . . . .	49
3.5. Concluding remarks . . . . .	49
<b>4. Sub-wavelength Phononic Bandgap Opening in Bubbly Media</b>	<b>53</b>
4.1. Introduction . . . . .	53

4.2. Problem formulation and preliminary results . . . . .	54
4.3. Sub-wavelength bandgaps . . . . .	55
4.3.1. The asymptotic behavior of $\omega_1^\alpha$ . . . . .	56
4.3.2. Dilute case . . . . .	61
4.4. Numerical illustrations . . . . .	62
4.5. Multipole expansion method . . . . .	63
4.6. Concluding remarks . . . . .	67
<b>5. Sub-wavelength Focusing of Acoustic Waves in Bubbly Media</b> . . . . .	<b>69</b>
5.1. Introduction . . . . .	69
5.2. Single bubble . . . . .	70
5.2.1. The breathing approximation . . . . .	71
5.3. System of N bubbles . . . . .	74
5.3.1. The point scatterer approximation for N-bubbles . . . . .	75
5.4. Numerical illustrations . . . . .	76
5.4.1. System parameters . . . . .	76
5.4.2. <i>A priori</i> evaluation of focusing performance via the imaginary part of the Green's function . . . . .	76
5.4.3. Simulating wave propagation in a bubbly medium . . . . .	78
5.5. Concluding remarks . . . . .	83
<b>6. Double-negative Acoustic Metamaterials</b> . . . . .	<b>87</b>
6.1. Introduction . . . . .	87
6.2. Preliminaries . . . . .	89
6.2.1. Capacitance coefficients . . . . .	90
6.3. Resonance for a dimer consisting of two identical bubbles . . . . .	91
6.4. The point scatterer approximation of bubbles . . . . .	97
6.5. Homogenization theory . . . . .	101
6.5.1. The homogenized equations . . . . .	104
6.5.2. Double-negative refractive index media . . . . .	106
6.6. Numerical illustrations . . . . .	106
6.7. Concluding remarks . . . . .	108
<b>7. Sub-wavelength Localized Modes for Acoustic Waves in Bubbly Crystals With a Defect</b> . . . . .	<b>109</b>
7.1. Introduction . . . . .	109
7.2. Preliminaries . . . . .	111
7.3. Integral representation for bubbly crystals with a defect . . . . .	113
7.3.1. Bubbly crystals with a defect: problem formulation . . . . .	113
7.3.2. Effective sources for the defect . . . . .	114
7.3.3. Characterization of the effective sources . . . . .	114
7.3.4. Floquet transform of the solution . . . . .	117
7.3.5. The integral equation for the effective sources . . . . .	119

7.4. Sub-wavelength localized modes . . . . .	119
7.4.1. The resonant frequency of the localized mode . . . . .	119
7.4.2. Proof of Lemma 7.4.2 . . . . .	122
7.5. Numerical illustrations . . . . .	124
7.6. Concluding remarks . . . . .	126
<b>Appendices</b> . . . . .	<b>129</b>
<b>A. Appendix</b> . . . . .	<b>131</b>
A.1. Asymptotic expansions . . . . .	131
A.1.1. Asymptotic expansions in three dimensions . . . . .	131
A.1.2. Asymptotic expansions in two dimensions . . . . .	133
A.1.3. Asymptotic expansions in three dimensions for quasi-periodic op- erators . . . . .	135
A.2. Subwavelength bandgap opening in two dimensions . . . . .	135
A.3. Proof of Lemma 5.2.1 . . . . .	136
<b>Bibliography</b> . . . . .	<b>139</b>