

Contents

Abstract (English/Français/Deutsch)	vii
1 Introduction	1
1.1 Knapsack problems	2
1.2 Clustered planarity	4
1.3 Our results and organization of the chapters	4
2 A PTAS for the time-invariant incremental knapsack	7
2.1 Overview of basic techniques	9
2.2 A PTAS for IIK	10
2.2.1 Reducing IIK to special instances and solutions	10
2.2.2 A disjunctive relaxation	13
2.2.3 Rounding	16
2.3 Generalizations	21
3 An improved disjunctive relaxation for the min-knapsack	25
3.1 Overview of the technique	25
3.2 The disjunctive relaxation	27
4 On bounded pitch inequalities for the min-knapsack	31
4.1 Basics	31
4.2 A weak separation oracle for pitch-1 and pitch-2 inequalities	33
4.2.1 Restricting the set of valid pitch-2 inequalities	33
4.2.2 An oracle	35
4.2.3 Separating inequalities of pitch-3 and larger, with fixed support	37
4.3 Integrality gap for MINKNAP with bounded pitch inequalities	38
4.3.1 When $p=c$	38
4.3.2 CG closures of bounded rank of the natural MINKNAP relaxation	39
4.3.3 When all knapsack cover inequalities are added	41
4.3.4 When all bounded pitch and knapsack cover inequalities are added	42
5 Clustered planarity testing	45
5.1 Basic definitions and an overview of results	45
5.2 Algorithm	48

5.3	Weak Hanani–Tutte for two-clustered graphs	51
5.3.1	Proof of Theorem 26	52
5.4	Strong Hanani–Tutte for two-clustered graphs	53
5.4.1	Proof of Theorem 27	54
5.5	Strong Hanani–Tutte for c -connected clustered graphs	58
5.6	Counterexample on three clusters	59
5.6.1	Proof of Theorem 29	64
5.7	Small faces	65
5.7.1	Proof of Theorem 30	67
5.8	Concluding remarks	68
6	Future directions	71
A		73
A.1	Background on disjunctive programming	73
A.2	IIK, MKP, and UFP	73
	Bibliography	80
	Curriculum Vitae	81