

# Contents

<b>Acknowledgements</b>	<b>v</b>
<b>Abstract (English/Français)</b>	<b>ix</b>
<b>List of publications</b>	<b>xiii</b>
<b>Acronyms</b>	<b>xix</b>
<b>Introduction</b>	<b>1</b>
<b>1 Fundamental concepts of optical microresonators and soliton-based Kerr frequency combs</b>	<b>3</b>
1.1 Optical frequency combs . . . . .	3
1.1.1 Introduction . . . . .	3
1.1.2 Self-referencing and stabilization of optical frequency combs . . . . .	5
1.1.3 Applications of frequency combs . . . . .	6
1.1.4 Comb generation methods . . . . .	7
1.2 Microresonator-based optical Kerr frequency combs . . . . .	8
1.2.1 Fundamental concepts of nonlinear optics in waveguides . . . . .	9
1.2.2 Dielectric resonators . . . . .	14
1.2.3 Theory of microresonator-based optical frequency combs . . . . .	26
1.2.4 Dissipative Kerr solitons . . . . .	31
1.2.5 Applications of microresonator based combs . . . . .	37
<b>2 Experimental generation, probing and characterization of solitons</b>	<b>39</b>
2.1 Experimental soliton generation . . . . .	39
2.1.1 Resonator fabrication . . . . .	39
2.1.2 Resonator characterization . . . . .	41
2.1.3 Basic setup for soliton generation . . . . .	42
2.2 Probing the soliton parameters . . . . .	45
2.2.1 Non destructive probing with a network analyzer . . . . .	45
2.2.2 Soliton $\mathcal{S}$ -resonance . . . . .	49
2.3 Experimental scaling of the soliton properties . . . . .	53
2.3.1 Network analyzer-based digital locking of the detuning . . . . .	53
2.3.2 Detuning-dependent dissipative Kerr soliton duration . . . . .	54

2.3.3	Detuning-dependent mode crossings and soliton recoil . . . . .	57
<b>3</b>	<b>Soliton breathing and dynamical instabilities</b>	<b>63</b>
3.1	Detailed analysis of the fundamental soliton instabilities . . . . .	63
3.2	Breathing dissipative solitons . . . . .	68
3.2.1	Experimental identification of breathing . . . . .	70
3.2.2	Breathing dynamics . . . . .	71
3.2.3	Breathing region . . . . .	75
3.2.4	Real-time observation of breathers . . . . .	75
3.2.5	Breathers synchronizations . . . . .	78
3.3	Multi-soliton states switching . . . . .	80
3.3.1	Observation of switching between dissipative Kerr soliton states by laser backward tuning . . . . .	80
3.3.2	Deterministic switching of soliton states . . . . .	81
3.3.3	Thermally enabled transitions of soliton states . . . . .	83
3.3.4	Thermal lifting of the multi-soliton states degeneracy . . . . .	83
3.4	Intermode breathing . . . . .	86
3.4.1	Presentation and modeling . . . . .	86
3.4.2	Experimental results . . . . .	89
3.4.3	Mode crossing-induced multi-solitons breathing and destabilization . . . . .	92
<b>4</b>	<b>Microcomb stability and control</b>	<b>95</b>
4.1	Introduction . . . . .	95
4.2	Using the microresonator as an optical reference . . . . .	96
4.2.1	Improving the temperature stability . . . . .	96
4.2.2	Detuning stabilization using the soliton average power . . . . .	98
4.2.3	Detuning stabilization via offset Pound-Drever-Hall laser locking . . . . .	99
4.2.4	Dual-mode temperature compensation technique . . . . .	101
4.2.5	Summary . . . . .	103
4.3	Direct phase locking of the comb degrees of freedom . . . . .	104
4.3.1	Phase locking the pump to a reference comb . . . . .	104
4.3.2	Repetition rate actuation with a probe laser . . . . .	105
4.3.3	Stabilization results . . . . .	107
4.4	RF injection-locking for repetition rate control and microwave purification . . . . .	110
4.4.1	Stabilization of the repetition rate with the pump power and associated limitations . . . . .	110
4.4.2	Stabilization of the resonator and pump to a stable optical reference . . . . .	110
4.4.3	Repetition-rate injection-locking effect and locking range scaling . . . . .	111
4.4.4	Spectral purification effect . . . . .	115
4.4.5	Numerical modeling . . . . .	116
4.4.6	Experimental measurement of PM-to-PM transfer function . . . . .	116

<b>5</b>	<b>Ultralow-noise soliton microcomb for microwaves generation</b>	<b>119</b>
5.1	Introduction . . . . .	119
5.2	Soliton generation and stabilization with an ultra-low-noise pump laser . . . . .	120
5.2.1	Soliton generation procedure . . . . .	121
5.2.2	Optical phase lock loop . . . . .	121
5.2.3	Resonator stabilization . . . . .	122
5.3	Soliton repetition rate noise minimization . . . . .	122
5.3.1	Repetition rate dependence on detuning . . . . .	122
5.3.2	Coupling adjustment and impact on the repetition rate stability . . . . .	124
5.3.3	Soliton repetition rate quiet point . . . . .	125
5.3.4	Residual limitations . . . . .	126
5.4	Ultralow-noise microcomb as a broadband narrow laser reference . . . . .	128
5.5	Ultralow-noise photonic microwave synthesis using a soliton microcomb-based transfer oscillator . . . . .	130
5.5.1	Principle of the transfer oscillator technique . . . . .	131
5.5.2	Implementation with an auxiliary comb . . . . .	132
5.5.3	Result . . . . .	134
5.5.4	Division chain . . . . .	135
5.5.5	Outlook . . . . .	140
<b>6</b>	<b>Multiplexing solitons in a microresonator</b>	<b>143</b>
6.1	Introduction . . . . .	144
6.2	Spatial multiplexing with co-propagating pump fields . . . . .	146
6.2.1	Principle . . . . .	146
6.2.2	Identification of soliton resonances and tuning method . . . . .	146
6.2.3	First demonstration . . . . .	148
6.2.4	Repetition rate difference modification . . . . .	149
6.2.5	Intermodulation products . . . . .	152
6.3	Spatial multiplexing in the counter-propagating configuration . . . . .	156
6.3.1	Principle and demonstration . . . . .	156
6.3.2	Dual-comb spectroscopy . . . . .	157
6.3.3	Dual-comb imaging of the soliton dynamics . . . . .	157
6.3.4	Stability . . . . .	159
6.4	Triple soliton comb generation via spatial multiplexing . . . . .	160
6.4.1	Principle and demonstration . . . . .	160
6.4.2	Triple comb interferogram . . . . .	162
6.5	Summary and outlook . . . . .	165

<b>A</b>	<b>Additional experimental results and contributions</b>	<b>171</b>
A.1	Microwave-rates solitons in integrated $\text{Si}_3\text{N}_4$ microresonators . . . . .	171
A.1.1	Principle and sample description . . . . .	171
A.1.2	Phase noise characterization . . . . .	171
A.1.3	Soliton injection-locking . . . . .	175
A.1.4	Noise transduction mechanisms . . . . .	175
A.2	Self referencing of an on chip comb . . . . .	180
A.2.1	Broadband spectrum generation via soliton dispersive wave emission . . . . .	180
A.2.2	$2f - 3f$ interferometry and CEO detection . . . . .	180
A.2.3	Experimental setup for self-referencing . . . . .	181
A.2.4	Counting the continuous-wave pump laser frequency . . . . .	183
A.3	Pulsed driving of a crystalline microresonator . . . . .	185
<b>B</b>	<b>Experimental methods</b>	<b>187</b>
B.1	Laser phase noise characterization . . . . .	187
B.2	Frequency counter benchmark for Allan deviation measurement . . . . .	187
B.2.1	Allan variance . . . . .	188
B.2.2	Experimental results . . . . .	189
<b>C</b>	<b>Derivation of the coupled mode equations model</b>	<b>195</b>
	<b>Bibliography</b>	<b>222</b>
	<b>Curriculum Vitae</b>	<b>227</b>