
Contents

Contents	i
1 Introduction	3
1.1 Classical computation	3
1.2 Quantum computation	5
1.3 Overview of this thesis	7
2 Review of Quantum Information Processing	9
2.1 Basic Quantum Mechanics	9
2.1.1 Quantum bit	9
2.1.2 Operations	10
2.1.3 Quantum Circuits	11
2.1.4 Measurements	12
2.2 Error Correction	12
2.2.1 Channels and Noise Models	13
2.2.2 Repetition Code	14
2.2.3 Stabilizer Codes	17
2.2.4 The Threshold Theorem	19
2.2.5 Topological error correction	19
2.3 Braiding	23
2.3.1 Movement of Defects	24
2.3.2 CNOT using Braiding	25
2.3.3 Single Qubit Operators	26
2.4 High level overview	27
2.4.1 Quantum Compiler	27
3 Lattice Surgery Translation for Quantum Computation	29
3.1 Introduction	29
3.1.1 Toric Code	30
3.1.2 Surface Code	30

3.1.3	Planar Code	31
3.1.4	ICM Representation	32
3.1.5	Outline of the paper	33
3.2	Quantum Computing	34
3.3	Computation using Lattice Surgery	35
3.3.1	Merge operations	36
3.3.2	Split operations	37
3.3.3	Multi-target CNOT	38
3.3.4	Split and Merge circuit	39
3.4	ICM Representation	41
3.4.1	Inverse ICM	42
3.5	Implementation of I and C	44
3.5.1	Classical Algorithm	44
3.5.2	Lattice surgery translation	45
3.6	Stabilizer matrix	46
3.6.1	Merge Operation	47
3.6.2	Split Operation	48
3.6.3	Example algorithm	48
3.7	Measurement step	49
3.7.1	Special Cases Merge	50
3.7.2	Encoding	51
3.8	Examples	51
3.8.1	Steane code for $ Y\rangle$ -state-distillation	52
3.8.2	Efficiency of the distillation	55
3.8.3	Stabilizer Matrix Calculation	55
3.8.4	Reed-Muller code for $ A\rangle$ -state-distillation	56
3.8.5	Bravyi-Haah code for $ A\rangle$ -state distillation	56
3.9	Conclusion	59
4	Optimization of Lattice Surgery is NP-Hard	63
4.1	Introduction	63
4.1.1	Revision of Lattice Surgery	65
4.2	Optimality	66
4.2.1	(Non-physical) Problem Description	66
4.3	Proof of NP-completeness	69
4.3.1	Mapping	69
4.4	Discussion	72
4.5	Conclusion	73
4.6	Acknowledgments	73
5	Time versus Hardware: Reducing Qubit Counts with a (Surface Code) Data Bus	75
5.1	Introduction	75
5.2	Results	76

5.3	Methods	80
5.3.1	Does the data bus have the intended effect?	80
5.3.2	Do arbitrary length parity-check operators accumulate errors?	82
5.3.3	How high is the time overhead?	83
5.3.4	Computation-specific analysis	84
5.3.5	Counter-example	87
5.4	Discussion	88
5.5	Appendix	89
5.5.1	Explicit calculation of mixing different bases	89
5.5.2	Explicit calculation of the Y -state measurement	91
5.5.3	Rotated patches and their layout	92
5.5.4	Intuition behind the trade-off plots	92
5.5.5	Approximations for the worst case	94
6	Lattice Surgery on the Raussendorf Lattice	97
6.1	Introduction	97
6.2	Brief Review	98
6.2.1	Graph States	98
6.2.2	Measurement-Based Quantum Computation	99
6.2.3	The Raussendorf Lattice and the Surface Code	100
6.2.4	Planar code	100
6.3	Lattice Surgery on the Raussendorf Lattice	102
6.3.1	Boxes inside the Raussendorf Lattice	104
6.3.2	Logical Pauli-operators	106
6.3.3	Initialization and Measurement	106
6.3.4	Merges and Splits	107
6.3.5	Encoding States	108
6.3.6	Hadamard	111
6.3.7	Hadamard and ICM	114
6.4	Conclusion	114
6.5	Acknowledgments	115
7	A local and scalable lattice renormalization method for ballistic quantum computation	117
7.1	Introduction	118
7.2	Background	119
7.2.1	Graph States	119
7.2.2	Creation of the lattice	120
7.2.3	Error correction	121
7.3	Graph purification: General Idea	124
7.4	Implementation	125
7.5	Results	126
7.6	Drawbacks of this Method	128

CONTENTS

7.7	Possible Improvements	131
7.8	Workflow	132
7.9	Conclusion	132
7.10	Acknowledgements	135
8	Conclusion and Outlook	137
	Bibliography	141